[Goyal* *et al.*, 5.(5): May, 2016]
IC™ Value: 3.00

ISSN: 2277-9655
Impact Factor: 3.785

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

APPLICATION OF SIMULTANEOUS FEEDBACK CONTROLLER FOR FREE AND FORCED ACTIVE VIBRATIONS CONTROL

Rachin Goyal*, Amresh Kumar, Manoj Kumar
* Associate Professor, Chandigarh Engineering College, Landran, Mohali, India
Assistant Professor, Chandigarh Engineering College, Landran, Mohali, India
Professor, Chandigarh Engineering College, Landran, Mohali, India

DOI: 10.5281/zenodo.51854

ABSTRACT

Evergrowing customer demands and thus the manufacturing methods have become the backbone of global competition for a wide range of products. This has led to the development of intelligent, compact and lightweight smart machine tools. A great difficulty is faced in the design of such machine tools due to the fact that the reduction in weight results in low rigidity and poor vibration control characteristics. The induced vibrations in the system may destabilize the system and result in complete failure of the system. On the other hand, the successful vibrations reduction of the smart machine tools during machining process can improve productivity, increase quality, and reduce tool wear. Also healing the vibration error can reduce industrial waste, save money and improve design flexibility of new cutting tools. Therefore, there is a need to develop machine tools that are equipped with suitable vibration control features. Smart machine tools, technology using active control system may provide a solution to this problem. These machine tools use piezoelectric materials, electro-rheological fluids and shape memory alloys as sensors and actuators for providing effective vibration control.This paper presents the design and application of simultaneous feedback controller for controlling the free and forced active vibrations.

KEYWORDS: Simulateneous feedback controller, Free and forced Vibrations, Smart machine tools.

INTRODUCTION

It has been observed that the vibrations are produced in a machine tools due to cutting process itself rather when exciting forces come from outside elements. These vibrations are termed as self-induced vibrations or machine tool chatter. Many methods have been devised and used to dampen out their impact, using cork slabs, rubber pads and steel springs etc. or by increasing the rigidity, till now. This results in the increase in weight of machine tools which leads to high initial investment and more space utilization etc. A great difficulty is also faced in the design of such machine tools due to the fact that the reduction in weight results in low rigidity and poor vibration characteristics. Unless the vibration is effectively controlled, it may destabilize the system and may, very often, result in complete failure of the system. Therefore, there is a need to develop machine tools that are equipped with suitable vibration control features. Smart machine tools technology using active control system may provide a solution to this problem. These machine tools use piezoelectric materials, electro-rheological fluids and shape memory alloys as sensors and actuators for providing effective vibration control.

In this paper, a built-up structure made by joining two beams to form an inverted L – structure has been discussed. Piezoelectric materials, bonded on a part of the surface of the structure, have been used as sensors and actuators. For controlling the vibrating structure using piezoelectric materials, the dynamics of these structures must be known. The relationship of electromechanical coupling of piezoelectric materials with the dynamics of these structures had been established by using finite element methods. Modal parameters, i.e. frequencies, damping ratios and mode shapes of inverted L – structure were calculated using the finite element model. Any flexible, vibrating system can be modeled into state space form using modal parameters.

Active vibration control is a feedback type of control. A feedback controller uses the measurements taken from the system to decide the inputs, which are given to control the system. The performance of the control system may be specified in terms of settling time, peak overshoot, steady state error and peak voltage required for actuators.

There are several techniques available, such as root locus method, bode plot technique, pole placement, state feedback, etc which can be used to design a controller satisfying these specifications. This forms the backbone of classical control theory. The principle tool in all these techniques is the Laplace Transformations. This approach makes extensive use of the concept of transfer functions. Pole placement method has been used for the controller design and finally the optimization has been done to finalize the position of closed loop poles to design simultaneous feedback controller.

DESCRIPTION OF STRUCTURE

In this paper, a built-up structure made by joining two beams to form an inverted L shaped structure has been studied. It is a simple extension of a one-dimensional beam to a two-dimensional beam structure. The practical applications of this structure are included in robots and machine tools. For example, SCARA robot and radial drilling machine is representative of an inverted L structure. This structure is also extensively used in flexible spacecrafts. Piezoelectric materials, bonded on a part of the surface of the structure, have been used as sensors and actuators. A Schematic View of the Inverted L – Structure as shown in figure 1.

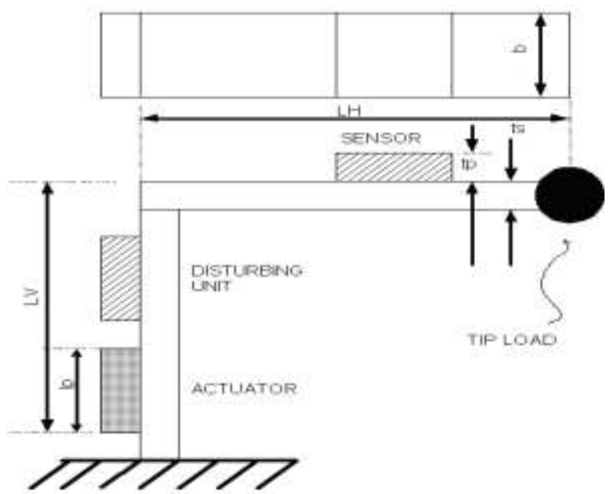

Fig. 1 Schematic View of the Inverted L – Structure

Table 1 Geometrical and Mechanical Properties

Dimension/Property	Material	
	STEEL	PZT
Length of Horizontal limb (mm)	L_H= 100	-------
Length of Vertical Limb (mm)	L_V=100	-------
Thickness (mm)	t_s=1	t_p=1
Length (mm)	l_s=20	l_p=20
Width (mm)	B=10	b=10
Young's Modulus (MPa)	E_s=210	E_p=64
Density (Kg/m^3)	ρ_s=7800	ρ_p=7650

ISSN: 2277-9655
Impact Factor: 3.785

Table 2 Electrical Properties of PZT

Property	Symbol	Value
Piezoelectric charge constant (m V^{-1})	d_{31}	171×10^{-12}
Piezoelectric charge constant (m V^{-1})	d_{32}	171×10^{-12}
Poisson's ratio	υ_p	0.28
Permittivity (Fm^{-1})	ε	106×10^{-12}

MATHEMATICAL MODELING

Finite element analysis is the fundamental tool for modeling these flexible structures. A large amount of literature is available till date for mathematical modeling of these structures using finite element techniques. For controlling the vibrating structures using piezoelectric materials, the dynamics of these structures must be known. Electromechanical coupling of piezoelectric materials can easily be related with the dynamics of these structures using finite element methods.

Using the matrix iteration method [1], the eigenvalue problem can be solved to give the natural frequencies and mode shapes of these structures. These parameters (i.e. natural frequencies and mode shapes) are the pre-requisites for designing an active vibration control system. Classical control systems are normally based on transfer functions. For multivariable systems with multi-inputs and multi-outputs, designer has to deal with matrices of transfer functions. In such cases, as the number of actuators and sensors are increased, the complexity of the control systems also increases. Thus for control of Multi-Input, Multi-Output systems, it is preferable to model the system in state-space form. Using the coupled control technique [2], the system can be written in state space form [3].

3.1 Lagrange's Equations of Motion for Linear Systems

The motion of a general linear system is given as [1]

$$\sum_{i,\,j=1}^{n}\left[m_{ji}\ddot{\Delta}_i(t)+c_{ji}\,\dot{\Delta}_i+k_{ji}\,\Delta_i(t)\right]=Q_j(t) \quad (1)$$

Where Δ_i (t) is the physical displacement, $\dot{\Delta}_i$ (t) is physical velocity and $\ddot{\Delta}_i$ (t) is the acceleration at time instant t for the particular degree of freedom i. The vector of externally applied forces is denoted by $\mathbf{Q}_j$ (t). Also m, c and k are the elements of mass, damping and stiffness matrices respectively. Equation (1) represents a set of n simultaneous second – order ordinary differential equations in generalized coordinates, and are called Lagrange's equations of motion. This relation approximates infinitely many degree-of-freedom distributed systems, by an n-degree of freedom system. This relation can be written in matrix form as

$$\mathcal{M}\,\ddot{\boldsymbol{\Delta}}(t)+\mathcal{C}\,\dot{\boldsymbol{\Delta}}(t)+\mathcal{K}\,\boldsymbol{\Delta}(t)=\mathcal{Q}(t) \quad (2)$$

Where **M**, **C** and **K** are the global mass, damping and stiffness matrices respectively, and **Q** (t) is the vector of physical applied forces at various degrees of freedom at instant of time t. The column vector $\boldsymbol{\Delta}(t)$ is called physical displacements at time t. These global matrices are obtained from finite element modeling of structure.

3.2 Finite Element Modeling

Discrimination of the continuous L-structure into finite number of segments has been shown in Figure 2 .Any framed structure, vibrating in a single plane can be assumed to be made of 2D beam or frame elements [4]. Each of these elements may be loaded with axial as well as transverse forces, along with bending moments. At each axial end of the element, called a node, there are horizontal and vertical deflections and slopes. The stiffness and mass matrices for such element would be of the size 6x6 [5]. These 2D beam elements can be horizontal, vertical or at an angle of inclination ‘α’. In case the element is mounted with a piezoelectric element, the mass and stiffness elements are modified [5]. Effective mass and flexural rigidity of L-Structure may be obtained by algebraically adding the masses and flexural rigidities of that of structure and attached PZT patches. The moment of Inertia (for flexural rigidity) of the element is taken about an axis perpendicular to the plane of the paper. Each element is having 6 degree of freedom as shown in figure 3.

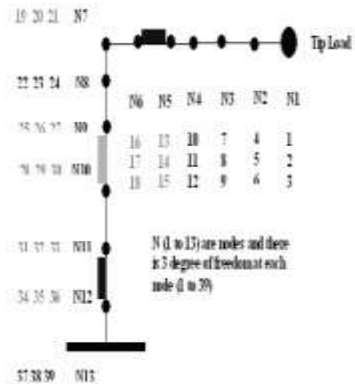

Fig.2 L-Structure (Discrete Form)

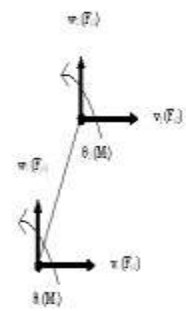

Fig.3 Six D.O.F. of 2D Beam Element

3.3 Natural Frequencies and Mode Shapes

The Lagrange’s Equation (2), If C = 0 (undamped), Q (t) = 0 (without applying force) leads to following matrix eigenvalue-eigenvector problem for the natural frequencies w and their corresponding mode shape vector X.

$$M^{-1} KX = \omega^2 \quad (3)$$

The natural frequencies, the square roots of the eigenvalues of $M^{-1}K$ are obtained by

$$\det |K - \omega^2 M| = 0 \quad (4)$$

It leads to *n*th order algebraic equation in ω^2 with real coefficients to give systems *n* natural frequencies.

3.4 Modal Analysis

The eigenvalue problem associated with undamped free-vibration system for relation (2) is

$$(K - \omega^2 M)\, \phi = 0 \quad (5)$$

Where the transformation matrix ϕ is known as mass normalized Modal matrix. Following transformations relates physical displacement ‘Δ’ to modal displacement ‘q’ at time instant t

$$\Delta (t) = q (t) \quad (6)$$

This mode superposition method is used to transform the ‘coupled equations of motion’ in physical co-ordinates to a set of ‘uncoupled equations of motion’ in the modal co-ordinates

$$\mathbf{\Phi}^{T}\mathcal{M}\,\mathbf{\Phi}\,\ddot{\mathbf{q}}(t)+\mathbf{\Phi}^{T}\mathcal{C}\,\mathbf{\Phi}\,\dot{\mathbf{q}}(t)+$$
$$\mathbf{\Phi}^{T}\mathcal{K}\,\mathbf{\Phi}\,\mathbf{q}(t)=\mathbf{\Phi}^{T}\mathcal{Q}(t) \qquad (7)$$

where

$\mathbf{\Phi}^{T} M\ \mathbf{\Phi}=\mathbf{I}$ is unity matrix

$\mathbf{\Phi}^{T} K\ \mathbf{\Phi}=\boldsymbol{\omega}^{2}$ is a diagonal matrices containing square of natural frequencies and

$\mathbf{\Phi}^{T}\mathcal{C}\ \mathbf{\Phi}=\tilde{\mathbf{C}}$ is a symmetric damping matrix.

In case of proportional damping, the normalized damping matrix is given as [6]

$$\tilde{\mathbf{C}} = \text{diag}[\ 2\ \xi_r\ \omega_r\ ] \qquad (8)$$

where ξ_r is the damping ratio associated with r'th particular mode. Thus, the uncoupled system of equations with proportional damping takes the following form

$$\ddot{\mathbf{q}}(t)+2\,\xi\,\boldsymbol{\omega}\,\dot{\mathbf{q}}(t)+\boldsymbol{\omega}^{2}\mathbf{q}(t)=\mathbf{\Phi}^{T}\mathcal{Q}(t) \qquad \mathbf{(9)}$$

the modal matrix $\mathbf{\Phi}$ for an N degree of freedom system may be written in component form as

$$\begin{bmatrix} {}^{(1)}\Phi_1 & {}^{(2)}\Phi_1 & \cdots & {}^{(N)}\Phi_1 \\ {}^{(1)}\Phi_2 & {}^{(2)}\Phi_2 & \cdots & {}^{(N)}\Phi_2 \\ \vdots & \vdots & \ddots & \vdots \\ {}^{(1)}\Phi_N & {}^{(2)}\Phi_N & \cdots & {}^{(N)}\Phi_N \end{bmatrix} \qquad (10)$$

Where ${}^{(j)}\Phi_k$ is the modal co-ordinate at k^{th} degree of freedom for j^{th} mode, and each column in the matrix represents the eigenvectors. The natural frequencies are collected to form the $\boldsymbol{\omega}^2$, diagonal matrix of modal frequencies squared, as

$$\boldsymbol{\omega}^{2}=\begin{bmatrix} \omega_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \omega_3^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \omega_N^2 \end{bmatrix} \qquad (11))$$

where ω_i is the natural frequency of the i^{th} mode and i=1 to N. For most of the structure systems under practical loading, only first few modes need to be considered. Thus the mode super position method is used to form a reduced order dynamic system [7]. Depending upon the number of modes to be considered, the dimensions of the system are changed from N to R where R (R<N) is the reduced order model of the complete system.

3.5 Modal State Space Control

It has been mentioned earlier that for a physical structure only the first few modes are important from vibration control considerations. If such a structure is controlled by using discrete distributed actuators numbering 'a' and discrete distributed sensors numbering's', the following relations may be written. For 'r' modes and 'a' actuators, equation (9) takes the form

$$\ddot{q}_k(t)+2\xi_k\omega_k\dot{q}_k(t)+\omega_k{}^2 q_k(t)=\sum_{i=1}^{i=a}\left[{}^{(k)}\Phi_i Q_i(t)\right]$$

$$k=1,2,....r \qquad (12)$$

where Q_i is the actual force acting at i^{th} degree of freedom and q_k is modal displacement at k^{th} mode. And the sensor output at the i^{th} degree of freedom, by the contribution of 'r' modes is given by

$$\Delta_i(t)=\sum_{k=1}^{k=r}\left({}^{(k)}\Phi_i \quad q_k(t)\right) i=1,2,...,s \qquad (13)$$

where ${}^{(k)}\Phi_i$ is the mode shape, at i^{th} degree of freedom and for k^{th} mode. Relation (12) for single actuator and single mode, in matrix form, is written as

where q(t), $\dot{q}(t)$ and $\ddot{q}(t)$ are the modal displacement, modal velocity and modal acceleration respectively. By

$$\begin{Bmatrix}\dot{q}(t)\\ \ddot{q}(t)\end{Bmatrix}=\begin{bmatrix}0 & 1\\ -\omega^2 & -2\xi\omega\end{bmatrix}\begin{Bmatrix}q(t)\\ \dot{q}(t)\end{Bmatrix}+\begin{Bmatrix}0\\ \Phi_{actuator}\end{Bmatrix}\{Q_{actuator}\}$$

making the substitution $w_1=q(t)$ and $w_2=\dot{q}(t)$ for the single particular mode, we may write the above equations as

$$\begin{Bmatrix}\dot{w}_1(t)\\ \dot{w}_2(t)\end{Bmatrix}=\begin{bmatrix}0 & 1\\ -\omega^2 & -2\xi\omega\end{bmatrix}\begin{Bmatrix}w_1(t)\\ w_2(t)\end{Bmatrix}$$

$$+\begin{Bmatrix}0\\ \Phi_{actuator}\end{Bmatrix}\{Q_{actuator}\} \qquad (14)$$

so that $w_1(t)$ is the modal displacement and $w_2(t)$ is the modal velocity. This type of representation is known as state space form. For the system having 'r' modes, equations (12) and (13) can be written in matrix state space form as

$$\dot{\mathcal{W}}(t)=\mathcal{F}\ \mathcal{W}(t)+\mathcal{G}\ \boldsymbol{u}(t)$$

$$\mathcal{y}(t)=\mathcal{H}\ \mathcal{W}(t) \qquad (15)$$

where modal state vector is defined as

$$w(t)=\begin{bmatrix}w_1(t)\\ w_2(t)\\ w_3(t)\\ w_4(t)\\ \vdots\\ w_{2r-1}(t)\\ w_{2r}(t)\end{bmatrix} \qquad (16)$$

Such that $w_{2r-1}=q_r \quad w_{2r}=\dot{q}_r$. Other matrices i.e. **F**, **G** and **H** are called system matrices [1].

SYSTEM TRANSFORMATIONS

4.1 Transformation from Continuous to Discrete Form

Where F and G and H are the system matrices in discrete form (whereas *F, G, H*, as found above, are system matrices in continuous form) and t is the sampling time. The state vector *W* is changed by W for the discrete system. In practice it is not necessary to solve these equations manually. There is a MATLAB function called **c2dm** that converts a given

continuous system (either in transfer function or state-space form) to discrete system using the zero-order hold operation explained above. The basic command for this c2dm (in state space form) is
[F, G, H, J] = c2dm (*F*, *G*, *H*, *J*, Ts, 'zoh')

4.2 Transformation from State Space to Difference Equation Form

For the design of pole placement control the systems model in difference equation form is more suitable [8]. Hence the state space model is converted into difference equation model. Using MATLAB command **ss2tf** the system in state – space form can be converted to transfer function form.

SIMULTANEOUS FEEDBACK CONTROLLER

It is observed that in practice structures are subjected to free vibrations as well as forced vibrations. The controller designed for any one particular type of situation are either ineffective for the other situation or give control instability. It is observed by simulations that the *control gains designed for free vibrations are almost ineffective for forced vibration response*. On the other hand, *if control gains corresponding to forced vibration are applied to free vibrations in the presence of sensor noise, instability occurs [9]*. So, different control gains should be designed and implemented for each situation. There should be a provision in the algorithm to judge the particular type of vibration and switch to the particular set of gains.

ALGORITHM FOR SIMULTANEOUS CONTROL

To implement the simultaneous controller, the flow chart shown in figure 4.4 is used. At large amplitudes, the weight age of free vibrations is high. An integer number **J** is defined such as **J**=1024. The simultaneous controller continues to work for **J** number of points. The sensor data is collected for this number of points in buffer 1 of the memory. To evaluate the performance of the controller for that period of time (or that region), it is divided into 4 sub-regions of 256 points each. The Fast Fourier Transform (denoted by Y's) is obtained and compared for each of 256 points. If the system is excited by free vibrations, the FB controller designed for free vibrations is quite effective and Y1 > Y2 > Y3 > Y4 will be the sequence. In case, the system is excited by forced vibration conditions, the controller will not be able for vibration attenuation. The vibration amplitude will certainly not decrease (i.e. Y1 = Y2 = Y3 = Y4). So, this criterion is sufficient to check the stability of the overall system under forced vibrations [10]. If this type of situation arises, controller switches to the other controller (designed for forced vibrations). In this way the loop continues till shut down is requested.

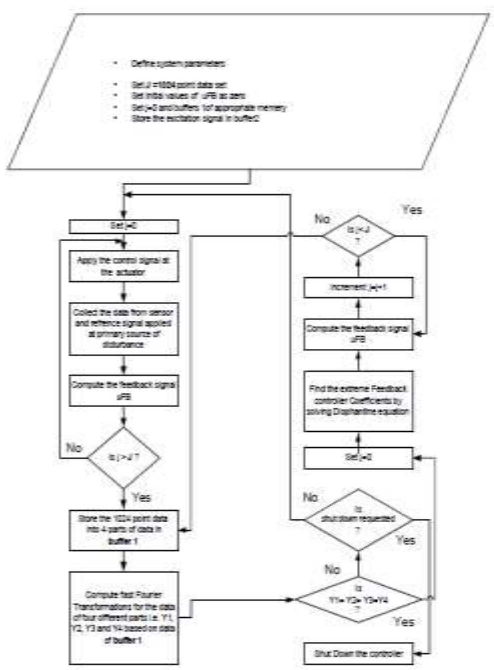

Fig. 4 Algorithm for Simultaneous Control

CONCLUSIONS

It is seen in the present work that separate sets of controller parameters (gains) should be applied for free and forced vibrations. The feedback controller designed for free vibrations is not effective for forced vibrations. On the other hand, the controller designed for forced vibrations makes the system unstable if applied to free vibrations. By combining both the controllers and applying the particular control gain for which it was designed, the closed loop system stability and high performance can be obtained. By optimizing the position of closed loop poles of the system, maximum reduction of vibrations is possible.

REFERENCES

[1] Meirovitch Gibbs, G. B. & Fuller, C. R. 1992, Experiments on active control of vibrational power flow using piezoceramic actuators/sensors', *AIAA Journal*, 30(2), pp. 457-463.
[2] Meirovitch, 1986, "*Dynamics and Control of Structures"*, McGraw Hill Publishing Company, NewYork.
[3] Chen, Li-Qun & Wei, Zhang. 2006, 'Adaptive vibration reduction of an axially moving string via a tensioner', International Journal of Mechanical Sciences, 48, pp. 1409- 1415.
[4] Chang, et al. 1992, *"Finite element analysis of composite structures containing distributed piezoceramic sensors and actuators",* AIAA Journal, 30(3), pp. 772-780.
[5] Kumar, R., Singh, S. P. & Chandrawat, H. N., 2005, "*Multivariable adaptive vibration control of smart structures using iterative (LQG) control strategies"*, Journal of Smart Materials and Structures, 14, pp. 953-962.
[6] Gibbs, G. B. & Fuller, C. R. 1992, 'Experiments on active control of vibrational power flow using piezoceramic actuators/sensors', *AIAA Journal*, 30(2), pp. 457-463.
[7] Kumar, R., 2007, *"Adaptive pole placement techniques for active vibration control of smart structures: A feasibility study",* Journal of Vibration and Acoustics, 129, pp. 602-615.
[8] Youn, S. H., Han, J. H.& Lee, I. 2000, 'Neuro-adaptive vibration control of composite beams subjected to sudden de-lamination', *Journal of Sound and Vibration*,238(2), pp. 215-231.
[9] Song Xubin, Mehdi Ahmadin, Steve Southward & Lane Miller, 2007, 'Parametric study of non-linear adaptive control algorithm for magneto-rheological suspension systems', *Communications in Nonlinear Science and Numerical Simulations,* 12, pp. 584-607.
[10] Wu JD, Jia-Hong Lin, 2005, 'Implementation of an active vibration controller for gear-set shaft using mu-analysis', *Journal of sound and vibration* , 281 pp. 1037-1055.